

## CE 3201: Introduction to Transportation Engineering Queuing Theory

Week Assigned: October 1, 2007

Due: Following week before lab

### **Objective:**

The assignment seeks to provide students with the opportunity to gain a better understanding of two queuing theories: M/D/1 and M/M/1.

### **Instructions:**

Be sure that you have downloaded the following two files from the class website:

<http://nexus.umn.edu/Courses/ce3201/LabHW2-1.xls>

<http://nexus.umn.edu/Courses/ce3201/LabHW2-2.xls>

You will be using these preformatted spreadsheets to simulate the two queuing models. These spreadsheets will help you answer the questions posed in each problem.

$$\text{Let } WT_q = \left( \frac{C_\lambda^2 + C_\mu^2}{2 * C_\lambda^2} \right) \left( \frac{\rho}{1 - \rho} \right) \frac{1}{\mu}$$

Where:

$WT_q$  Average customer delay in the queue

$C_\lambda$  Coefficient of variation (CV) of the arrival distribution

$C_\mu$  CV of the departure distribution

CV Standard deviation/mean; CV = (1/SqRt (mean)) for Poisson process and CV = 0 for constant distribution

$\mu$  Average departure rate

$\lambda$  Average arrival rate

$\rho$  Utilization = Arrival rate/service rate ( $\rho = \lambda/\mu$ )

### **Problem 1** (Simulating an M/D/1 queue):

*LabHW2-1.xls* is a preformatted spreadsheet for simulating an M/D/1 queue, i.e. a single-server queuing system with Poisson arrivals and constant service rates. As the arrival rate of a queuing system increases, the average delay experienced by users in the system also increases. Use the spreadsheet to simulate an M/D/1 queue with various demand levels:

Scenario	1	2	3	4	5	6	7	8	9	10
Arrival rate	.01	.025	.05	.1	.2	.3	.4	.5	.6	.7
Service rate	.5									

Since the simulation results are sensitive to the initial random seeds, it is required that each scenario be simulated with five different random seeds. Compute the average delay of all five runs for each level of the arrival rate (you need to execute 50 simulation runs to obtain the results for all ten scenarios).

Also, find the utilization for all ten scenarios. Based on the utilization and the distribution variability, use the above equation to compute the average delays for all scenarios with utilization values of **less than 1**.

Finally, summarize the average-delays obtained both from the simulation and from the  $WT_q$  equation in the same delay-utilization plot. Interpret your results. How does the average user delay change as utilization increases? Does the above equation provide a satisfactory approximation of the average delays?

**Problem 2** (Simulating an M/M/1 queue):

The second problem is the same as the first one, except that an M/M/1 queue will be simulated. That is the service rates are no longer constant but follow a Poisson distribution. You need to use *LabHW2-2.xls* this time. Simulate the M/M/1 queue with the following demand levels:

Scenario	1	2	3	4	5	6	7	8	9	10
Arrival rate	.01	.025	.05	.1	.2	.3	.4	.5	.6	.7
Service rate	.5									

Again, you need to use five different random seeds for each scenario. Summarize the results in a delay-utilization plot. Interpret your results (**You do NOT need to use the  $WT_q$  equation given above to compute delays for the M/M/1 queue**).

**Additional Questions**

1. Finally compare the M/M/1 queue and the M/D/1 queue. What conclusion can you draw? (For the same average arrival rate, do users experience the same delays in the two queuing systems? Why or why not?).
2. Provide a brief example where M/M/1 might be the appropriate model to use.
3. Provide a brief example where M/D/1 might be the appropriate model to use.