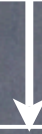


Trip Generation



Destination Choice



Mode Choice



Route Choice

Professor  
David  
Levinson

# Route Choice

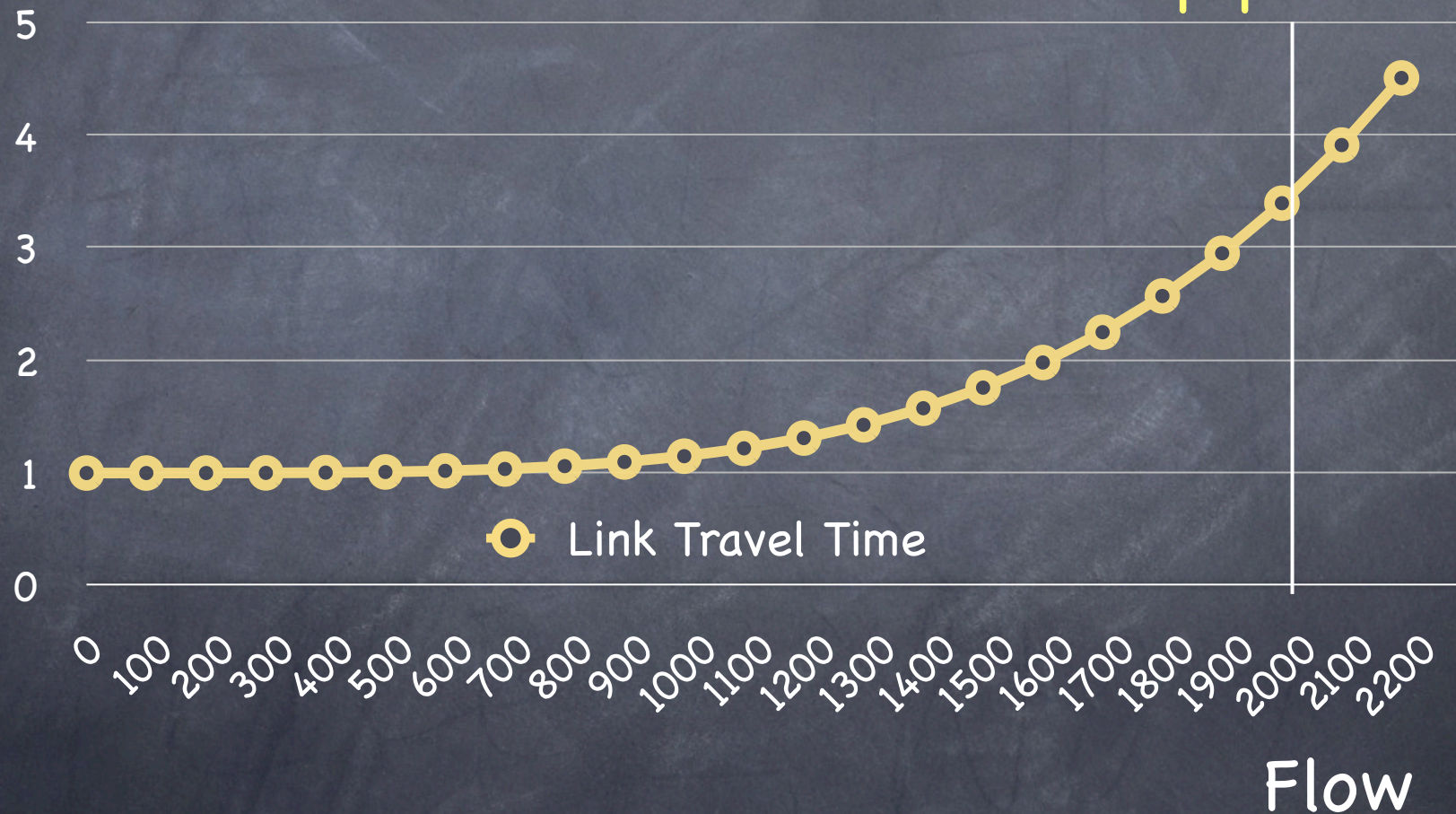
How travelers choose a route between A and B.

# Link Performance Function

Capacity = 2000

vplph

Travel Time



$$\text{travel time} = 1 + 0.15(\text{Flow}/\text{Capacity})^4$$

# Average and Marginal Cost

The cost that a driver imposes on others is called the marginal cost. However, when making decisions, a driver only faces his own cost (the average cost) and ignores any costs imposed on others (the marginal cost).

$$\text{Average Cost} = C_T / Q$$

$$\text{Marginal Cost} = dC_T / dQ$$

# Can Flow Exceed "Capacity"?

On a link, the capacity is thought of as "outflow."

Demand is inflow.

If inflow  $>$  outflow for a period of time, there is queueing (and delay).

For Example: For a 1 hour period, if 2100 cars arrive and 2000 depart, 100 are still there. This equation tries to represent that phenomenon in a simple way.

# Wardrop's Principles

## User Equilibrium

Each user acts to minimize his/her own cost, subject to every other user doing the same – Travel times are equal on all used routes and lower than on any unused route.

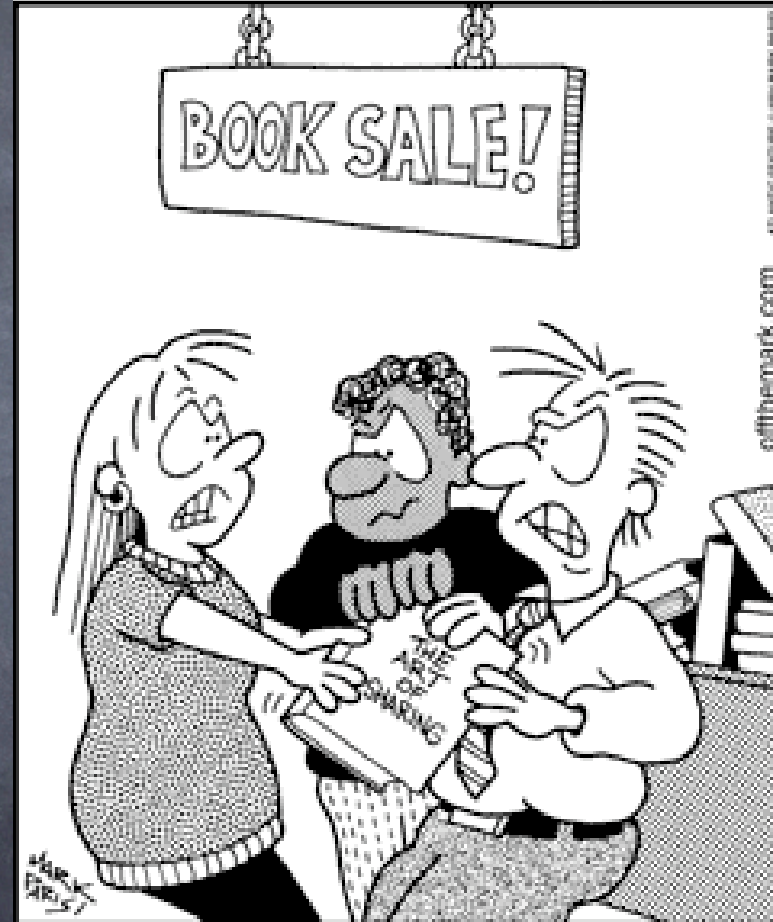
## System optimal

Each user acts to minimize the total travel time on the system.

People are selfish.

off the mark.com

by Mark Parisi



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# "Price of Anarchy"

The ratio of system-wide travel time under User Equilibrium and System Optimal conditions.

$$UE/SO > 1$$

For a two-link network with linear link performance functions (latency functions), Price of Anarchy is  $< 4/3$ .

# Is this too much?

Should something be done, or is 33% waste acceptable?

[The loss may be larger/smaller in other cases, under different assumptions, etc.]

# Conservation of Flow

An important factor in road assignment is the conservation of flow. This means that the number of vehicles entering the intersection (link segment) equals the number of vehicles exiting the intersection for a given period of time. (except for sources and sinks)

Similarly, the number of vehicles entering the back of the link equals the number exiting the front (over a long period of time).

# Example

Link 1



Link 2

Solve for the flows on Links 1 and 2 in the Simple Network just shown if the link performance function on link 1:

$$C_1 = 5 + 2 * Q_1$$

and the function on link 2:

$$C_2 = 10 + Q_2.$$

Time (Cost) is equal on all used routes so

$$C_1 = C_2$$

$$\text{And } Q_1 + Q_2 = Q_o = Q_d = 1000$$

$$5 + 2 * (1000 - Q_2) = 10 + Q_2$$

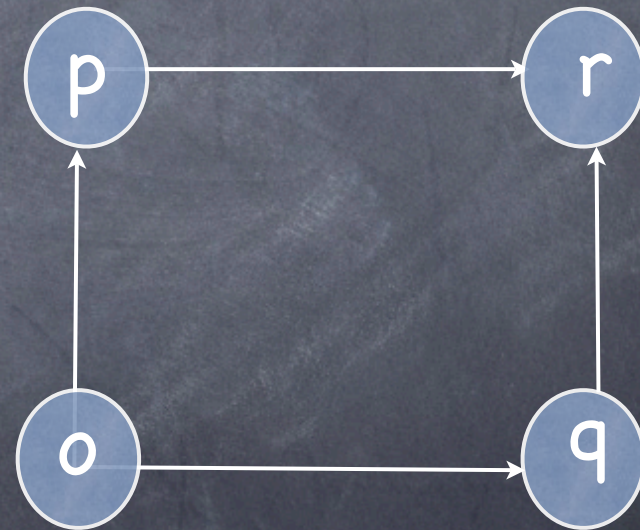
$$1995 = 3 Q_2$$

$$Q_2 = 665; Q_1 = 335$$

# Problem

- Example. Given a flow of six (6) units from origin "o" to destination "r". Flow on each route  $ab$  is designated with  $Q_{ab}$  in the Time Function. Apply Wardrop's Network Equilibrium Principle (Users Equalize Travel Times on all used routes)
- A. What is the flow and travel time on each link?  
(complete the table below) for Network A

Link	Time Function	Flow	Time
o-p	$5 * Q_{op}$		
p-r	$25 + Q_{pr}$		
o-q	$20 + 2 * Q_{oq}$		
q-r	$5 * Q_{qr}$		



B. What is the system optimal assignment?

C. What is the Price of Anarchy?

# Solution a: User Equilibrium

These four links are really 2 links O-P-R and O-Q-R, because by conservation of flow  $Q_{op} = Q_{pr}$  and  $Q_{oq} = Q_{qr}$ .

By Wardrop's Equilibrium Principle, The travel time (cost) on each used route must be equal. So

$$C_{opr} = C_{oqr}.$$

OR

$$25 + 6 * Q_{opr} = 20 + 7 * Q_{oqr}$$

$$5 + 6 * Q_{opr} = 7 * Q_{oqr}$$

$$Q_{oqr} = 5/7 + 6/7 Q_{opr}$$

Link	Time Function
o-p-r	$25 + 6 * Q_{opr}$
o-q-r	$20 + 7 * Q_{oqr}$

# Solution a: User Equilibrium

By the conservation of  
flow principle

$$Q_{oqr} + Q_{opr} = 6$$

$$Q_{opr} = 6 - Q_{oqr}$$

By substitution

$$Q_{oqr} = \frac{5}{7} + \frac{6}{7} (6 - Q_{oqr}) = \frac{41}{7} - \frac{6}{7} Q_{oqr}$$

$$\frac{13}{7} Q_{oqr} = \frac{41}{7}$$

$$Q_{oqr} = \frac{41}{13} = 3.15$$

$$Q_{opr} = 2.84$$

Check

$$42.01 = 25 + 6(2.84) \quad ? \quad 20 + 7(3.15) = 42.05 \quad \text{Check (within rounding error)}$$

Link	Time Function	Flow	Time
o-p	$5 * Q_{op}$	2.84	14.2
p-r	$25 + Q_{pr}$	2.84	27.84
o-q	$20 + 2 * Q_{oq}$	3.15	26.3
q-r	$5 * Q_{qr}$	3.15	15.75

# Solution b: System Optimal

What is the system optimal assignment for the previous example

$$\text{Conservation of Flow: } Q_{oqr} + Q_{opr} = 6$$

$$\text{Total Delay} = Q_{opr}(25 + 6 \cdot Q_{opr}) + Q_{oqr}(20 + 7 \cdot Q_{oqr})$$

$$25 Q_{opr} + 6 Q_{opr}^2 + (6 - Q_{opr})(20 + 7 Q_{opr})$$

$$25 Q_{opr} + 6 Q_{opr}^2 + 372 - 62 Q_{opr} - 42 Q_{opr} + 7 Q_{opr}^2$$

$$13 Q_{opr}^2 - 79 Q_{opr} + 372$$

TWO SOLUTION METHODS:

"Solver", "Analytic"

Analytic

Min Total Delay

$$d\text{Delay}/dQ = 26 Q_{opr} - 79 = 0$$

$$Q_{opr} = 79/26 = 3.04$$

$$Q_{oqr} = 6 - Q_{opr} = 2.96$$

# Price of Anarchy

User Equilibrium: Total Delay =  $42.01 * 6 = 252.06$

System Optimal: Total Delay =  $3.04(25 + 6 * 3.04) + 2.96(20 + 7 * 2.96) = 94.24 + 120.53 = 214.77$

Price of Anarchy =  $252.06 / 214.77 = 1.17 < 4/3$

# Thought Questions

How can we get drivers to consider their marginal cost?

Alternatively: How can we get drivers to behave in a "System Optimal" way?

Is past behavior reflective of future behavior?

Can the future be predicted?

Is the future independent of decisions, or are prophecies self-fulfilling?

How do we know if forecasts were successful?

Against what standard are they to be judged?

What values are embedded in the  
planning process?

What happens when values change?

# Questions

- Questions?

# Abbreviations

- VDF - Volume Delay Function
- LPF - Link Performance Function
- UE - User Equilibrium
- SO - System Optimal
- DTA - Dynamic Traffic Assignment
- DUE - Deterministic User Equilibrium
- SUE - Stochastic User Equilibrium
- AC - Average Cost
- MC - Marginal Cost

# Key Terms

- Route assignment, route choice, auto assignment
- Volume-delay function, link performance function
- User equilibrium
- System optimal
- Conservation of flow
- Average cost
- Marginal cost

# Variables

- $C_T$  - total cost
- $C_k$  - travel cost on link  $k$
- $Q_k$  - flow (volume) on link  $k$

# Inputs to Transportation Planning Models

- Measured Inputs:

- Travel diaries,
- land use,
- networks
- desire lines
- vehicle counts
- origin-destination surveys
- home interview surveys

- Forecast Inputs

- population,
- population distribution,
- per capita income,
- auto ownership,
- travel behavior (Origin-destination flows by purpose and mode)
- link speeds and flows by mode and time of day
- Emissions
- Change in land use as a result of network

# Issues and Alternatives

- Issues

- Inconsistencies, Estimation Bias
- Aggregation Error
- Decision Making Not Necessarily Sequential
- Mission Questions (Time of Day, Vehicle Ownership, Land Use, Network Expansion)
- Travel Time Consistency
- Detail, Realism

- Alternatives

- Choice Paradigm
- Activity Analysis Paradigm
- Simulation Paradigm
- Simultaneous Decision Making (Equilibrium) Paradigm

Questions Before Quiz 1?